MODELLING OF KINK-BAND GROWTH BASED ON THE GEOMETRICALLY NON-LINEAR THEORY

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Abstract
We propose a new and computationally efficient continuum damage based model, able to predict fibre/matrix shear failure under longitudinal compression for a UD ply. A structure tensor based continuum damage formulation is placed in context with the UD ply, where the elastic material response is governed by transverse isotropy. To represent the proper energy dissipation, an elastic damage model is formulated in the invariants of fibre/matrix shear and fibre compression, including failure initiation and progressive damage modeling. We are guided by the anisotropic elastic model to define four strain invariants, representing key features of the UD-ply microstructure. The damage model is applied to a Non-Crimp Fabric (NCF) composite and compared to a state of the art model based on kinking theory [6]. Instead of invoking the geometric instability into the material model, a key feature is to consider the geometrical fibre kinking instability on the macro-level based on a finite strain formulation.

1. Introduction

Bearing failure and crash simulations involving structural polymeric composites are a few examples where it is crucial to account for the longitudinal compressive response of Carbon Fibre Reinforced Plastics (CFRP). The ultimate goal is to facilitate the predictive modeling/simulation of 3D structural composites, where a major issue is to account for the intra-laminar failure mechanisms. Our understanding of these mechanisms, and in particular, longitudinal compressive failure has been under development for quite some time. The behavior of UD-plies subjected to compression induced fibre/matrix shearing has been studied experimentally and based on micromechanically motivated models. This mechanism is difficult to model since it involves both matrix-shear induced failure combined with fibre buckling within the same failure scenario. In particular, the micro-buckling (or fibre kink) phenomenon has been studied, cf. the work by [1], [2] and [3] with simplified matrix and fibre responses.

Later on it has been realized that the compressive failure mechanism is attributed to geometrical instability combined with fibre/matrix shearing under compression as induced by misaligned fibres, as discussed in ref. [4]; hence, the importance of modeling the degrading ply behavior under fibre/matrix shearing is underlined. However, for computational efficiency of 3D laminates failure modeling, phenomenological models that are micromechanically motivated are needed. In this context we mention refs [5] presenting models involving the fibre kink initiation. More recently, Gutkin et al. [6] presented a model accounting for fibre kinking in a mechanistic fashion.

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2. Continuum damage modeling of intra-laminar fracture in a UD ply

2.1. Representation of elastic transverse isotropy

To start with, let's consider the case of elastic transverse isotropy in the spirit of the St. Venant Kirchhoff model used for isotropic elasticity. To this end we are introducing the Lagrange strain tensor defined as $\boldsymbol{E} = \frac{1}{2}(\boldsymbol{C} - \mathbf{1})$, where $\boldsymbol{C} = \boldsymbol{F}^t \cdot \boldsymbol{F}$ is the right Cauchy Green deformation tensor and $\boldsymbol{F}$ is the deformation gradient. In terms of the Lagrange strain, we introduce ordinary and structure based strain invariants defined by

$$i_1 = \mathbf{1} : \boldsymbol{E} \, , \, i_2 = \mathbf{1} : \boldsymbol{E}^2 \, , \, i_2^{\mathrm{dev}} = i_2 + \frac{1}{3} i_1^2 \, , \, i_1^m = \boldsymbol{M} : \boldsymbol{E} \, , \, i_2^m = \boldsymbol{M} : \boldsymbol{E}^2 \, , \, i_2^{m,\mathrm{dev}} = i_2^m - \left(i_1^m\right)^2 \qquad (1)$$

where the structure tensor $\boldsymbol{M}$ describes the influence of the fiber direction in Fig. 1a by $\boldsymbol{M} = \boldsymbol{N} \otimes \boldsymbol{N}$.

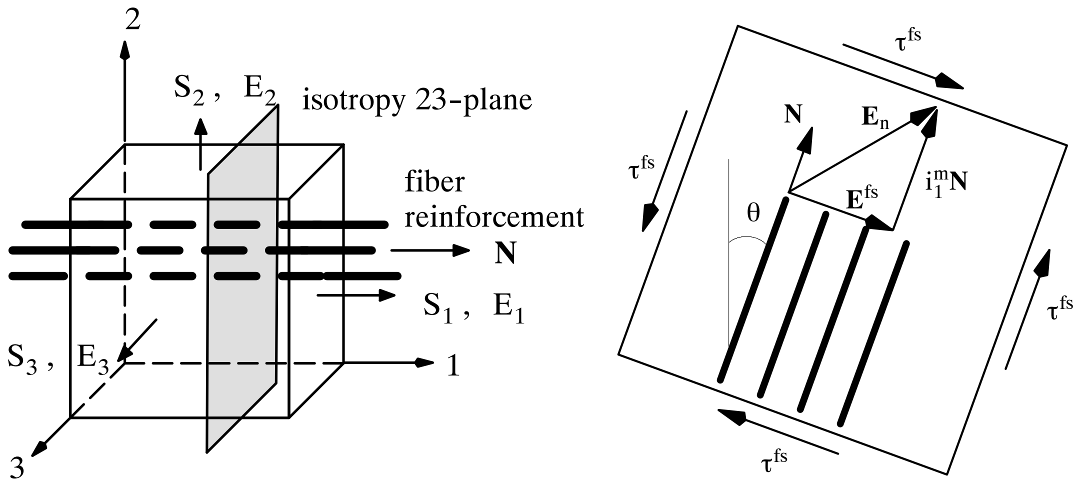

Figure 1. a) A UD laminate in transverse isotropy with fibre orientation $\boldsymbol{N}$. b) Fibre/matrix shear stress τ^f associated with fibre/matrix shear strain γ^f.

To model the elastic transverse isotropy of the UD ply, let us consider the effective free energy $\hat{\psi}$ per unit volume effective material, subdivided into the four different strain energy contributions

$$\hat{\psi} = \hat{\psi}^{\mathrm{dev}} + \hat{\psi}^{\mathrm{vol}} + \hat{\psi}^{\mathrm{fs}} + \hat{\psi}^{f} \qquad (2)$$

where the deviatoric (or shape distortion) and the volume change energies are represented by $\hat{\psi}^{\mathrm{dev}}$, $\hat{\psi}^{\mathrm{vol}}_{+}$ and $\hat{\psi}^{\mathrm{vol}}_{-}$. These are formulated in terms of the strain invariants as

$$\hat{\psi}^{\mathrm{dev}} = \frac{1}{2} 2G i_2^{\mathrm{dev}} \, , \, \hat{\psi}^{\mathrm{vol}} = \hat{\psi}^{\mathrm{vol}}_{-} + \hat{\psi}^{\mathrm{vol}}_{+} = \frac{1}{2} K \left(< i_1 >_{-}^{2} + < i_1 >^{2} \right) \qquad (3)$$

where the shear modulus G and the bulk modulus K are material parameters of the matrix material. A feature of the model is that no damage is associated with hydrostatic compression $< i_1 >_{-} \neq 0$, whereby we consider the volumetric portion of the free energy subdivided.

In order to represent the fibre reinforcement, we introduce the elastic constitutive relations for fibre strain and shear, respectively, as

$$\sigma^f = E^f \epsilon^f, \tau^f = G^f \gamma^f \text{ with } \gamma^f = \sqrt{2 i_2^{m,\mathrm{dev}}} \qquad (4)$$

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where σ^f is the *normal* fibre stress, ϵ^f is the fibre strain, τ^f is the fibre *shear* stress and γ^f is the shear strain of the fibres, cf. Fig. 1b. Here, E^f and G^f are material parameters representing the normal and shear fibre stiffness. As to the stored free energies due to fibre/matrix shear and normal fibre straining, denoted ψ^{fs} and ψ^f respectively, actions we postulate

$$\psi^{\mathrm{fs}} = \frac{G^f}{2}\left(\gamma^f\right)^2 = \frac{1}{2}\frac{\left(\tau^f\right)^2}{G_f}\ ,\ \psi^f = \frac{1}{2}E^f\left(\epsilon^f\right)^2 = \frac{1}{2}\frac{\left(\sigma^f\right)^2}{E^f} \tag{5}$$

Please note that in the formulation of the fibre strain there is generally a Possion effect associated with the fiber direction. This Possion effect is described by ν^f whereby the fibre strain can be related to the strain invariants as

$$\epsilon^f := \frac{\left(1-2\nu^f\right)i_1^m + \nu^f i_1}{(1-2\nu^f)(1+\nu^f)} \tag{6}$$

2.2. Representation of intra-laminar damage degradation in compression

Based on the considered strain invariants of the ply, a continuum damage model emphasized on representing degradation in compression is proposed using the isotropic damage variable α. We then introduce the damage function $f[\alpha]$ for the degradation of the matrix and fibre/matrix shear responses and the function $g[\alpha]$ representing the compressive fibre response due to crushing. These functions are defined in terms of a degradation $1-\alpha$ raised to the exponents β^f and β^g defined as

$$f[\alpha] = (1-\alpha)^{\beta^f}\ ,\ g = (1-\alpha)^{\beta^g} \tag{7}$$

Please to note that the rate of the degradation becomes different depending on the β^f- and β^g–exponents. In particular, the rate of damage degradation is much faster in the matrix and fibre/matrix shear responses as compared to the fibre crushing in the beginning of the damage progression.

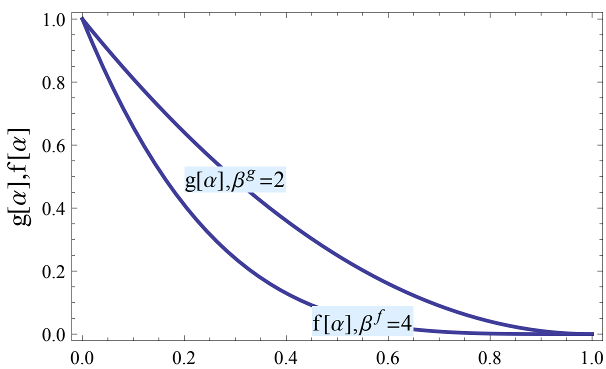

Figure 2. Damage degrading functions $f[\alpha]$ and $g[\alpha]$.

In view of the discussed scenario for the degradation of the UD fibre reinforced material, we formulate the degraded free energy function as

$$\psi = f[\alpha]\left(\hat{\psi}^{\mathrm{dev}} + \hat{\psi}_+^{\mathrm{vol}} + \hat{\psi}^{\mathrm{fs}}\right) + g[\alpha]\hat{\psi}^f + \hat{\psi}_-^{\mathrm{vol}} \tag{8}$$

The rate of energy dissipation per unit volume due to the damage variable is $\mathcal{D} = \boldsymbol{S} : \boldsymbol{E} - \psi \geq 0$, corresponding to the state equation of the 2nd Piola Kirchhoff stress

$$\boldsymbol{S} = \frac{\partial\psi}{\partial\boldsymbol{E}} = f[\alpha]\left(\hat{\boldsymbol{S}}^{\mathrm{dev}} + \hat{\boldsymbol{S}}_+^{\mathrm{vol}} + \hat{\boldsymbol{S}}^{\mathrm{fs}}\right) + g[\alpha]\hat{\boldsymbol{S}}^f + \hat{\boldsymbol{S}}_-^{\mathrm{vol}} \tag{9}$$

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and the reduced dissipation $\mathcal{D} = A\dot{\alpha} \geq 0$. Here, the damage driving force A now becomes

$$A = -f'[\alpha]\left(\hat{\psi}^{\text{dev}} + \hat{\psi}_{+}^{\text{vol}} + \psi^{\text{fs}}\right) - g'[\alpha]\psi^{f} \tag{10}$$

2.3. A local damage evolution model

Upon introducing the dissipation potential due to the diffuse crack surface propagation is defined as

$$G = \int_{\mathcal{B}} g_c \gamma[\alpha] dV \tag{11}$$

where g_c is the crack surface energy release parameter (like the G_f^I in mode I fracture) and $\gamma = \gamma[\alpha]$ the crack surface density per unit volume. This defines the fracture surface area A_l as

$$A_l[\alpha] = \int_{\mathcal{B}} \gamma[\alpha] dV \tag{12}$$

We immediately find that the dissipation rate due to the crack surface propagation becomes

$$\dot{G} = \int_{\mathcal{B}} g_c \frac{\partial \gamma}{\partial \alpha} \dot{\alpha} dV \geq 0 \text{ with } \frac{\partial \gamma}{\partial \alpha} \geq 0 \text{ and } \dot{\alpha} \geq 0 \tag{13}$$

Evidently $\dot{G}$ represents the dissipation rate due to the *diffuse* crack evolution, confined within a internal width l, pertinent to our continuum damage type of fracture modeling. In order to consider the dissipation due to localized crack propagation lets consider a simple local damage loading function $\phi_\alpha[A, \alpha] \leq 0$ from the non-local one in the previous sub-section. We now write the loading function as

$$\phi_\alpha[A, \alpha] = A - \frac{g_c}{l}\alpha \tag{14}$$

whereby the dissipation becomes maximum in the sense that

$$\mathcal{D} = \sup_{A,\mu} A\dot{\alpha} - \mu\phi_\alpha = \frac{g_c}{l}\alpha\dot{\alpha} \geq 0 \text{ with } \mu = \dot{\alpha} \text{ and } A = \frac{g_c}{l}\alpha \tag{15}$$

corresponding to the Kuhn-Tucker conditions

$$\dot{\alpha} \geq 0, \phi \leq 0, \phi\dot{\alpha} = 0 \tag{16}$$

To arrive at model that provides mesh objective response, we find for the local damage model that the internal length paramter is related to the element diameter as $d_e = 2l$; please note that the progressive damage model is now associated with an assumed localized damage field, which in the finite element application is confined to the size of one element diameter d_e.

3. Numerical verification example

The damage model is applied to a Non-Crimp Fabric (NCF) composite and compared to the one in ref. [6], based on kinking theory. The evolution of the damage variable is found from an in-plane shear test and a uniaxial compression with test with $\theta = 5^0$ performed on a unidirectional laminate; the resulting model response after calibration is shown in Fig 3ab. We also included the model response in uniaxial compression for a range of off–axis angles θ, cf. Fig. 3b. The calibration procedure yields the classical material parameters pertinent to the effective elastic stress response as: $E_1 = 128000\frac{N}{mm^2}$, $E_2 = 7900\frac{N}{mm^2}$, $G_{12} = 3000\frac{N}{mm^2}$, $\nu_{12} = .3$, $\nu_{21} = \nu_{12}\frac{E_2}{E_1}$, and for the damage response: $g_c = 118\frac{N}{mm}$, $l = 4mm, \beta^g = 6.5, \beta^f = 9$. As to the adopted material parameters, a direct conversion from the presently introduced five tensor based parameters (ν^f, E^f, G^f, G and K) to the classical ones of the UD ply is employed. Interestingly, we find for realistic data with $\nu_{23} \to 0.4$ and $\nu_{12} \to 0.3$ that $\nu^f \to 0.0012 \approx 0$, whereby $\epsilon^f \approx i_1^m$ in practice. We also note that $E^f \approx E_1$ with these data.

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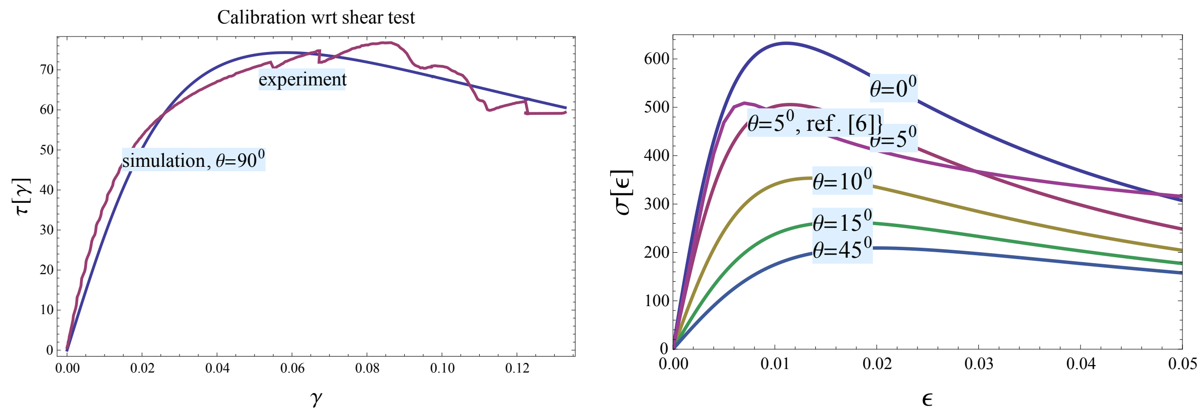

Figure 3. a) Model calibration result with respect to an in-plane pure shear test performed on a unidirectional laminate. b) Uniaxial compressive response for different fibre kinking of the present model compared to a kinking model developed in [6].

4. Conclusion

We have proposed a new and computationally efficient continuum damage based model that is able to predict the fibre/matrix shear response under compression. Two invariants are associated with isotropic matrix behavior and two the others are associated with the fibre/matrix shear and fibre straining. A Poisson coupling effect between the matrix and fibre responses is identified and modelled in the stored fibre energy. Our formulation is related to the CDM modeling adopted in e.g. refs. [7], [8], but without any damage–plasticity coupling for efficiency and simplicity. The damage models are applied to a Non-Crimp Fabric (NCF) composite and compared to a state of the art model based on kinking theory [6]. The results obtained so far indicate predictive capability as well as computational efficiency of the formulation. Instead of invoking the geometric instability into the material model, a key feature is to consider the geometrical fibre kinking instability on the macro-level based on a finite strain formulation.

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