QUASI-STATIC DELAMINATION MODEL FOR LAYERED ANISOTROPIC MATERIAL STRUCTURES

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Keywords: Anisotropic material, Symmetric Galerkin BEM, Delamination, Interface damage, Quasi-static delamination, Cohesive contact

Abstract
Mathematical model for numerical solution of damage in layered anisotropic material structures has been developed. In the model, various types of layered materials can be proposed by using any type of anisotropic materials of the layer with different mechanical properties. The analysis calculates stress and strain quantities in the structure and at the damaged interface. The concept of solution is based on quasi-static rate-dependent evolution of the delamination process at the interface. The proposed mathematical approach is based on an energetic formulation looking for a kind of weak solution. The solution is approximated by a time stepping procedure, the Symmetric Galerkin Boundary Element Method (SGBEM), and it utilizes non-linear programming algorithms. The acquired results are also presented, demonstrating their significance and application in engineering practice.

1. Introduction

Composite materials are commonly used as structural materials in lightweight structures at present. In aerospace applications, where a high strength-to-weight ratio is required and lightweight is a key aspect of the design, their level of structural responsibility has significantly increased as they are massively used in primary structures. Composite materials especially layered materials are frequently corrupted between the layers. From this point of view the availability of reliable methods for predicting the failure of these materials is perhaps one of the most important requirements when designing composite components and structures [3, 5].
However, predictions of initiation and propagation of damage in composites are still not sufficiently accurate and reliable, which leads to high safety coefficients in design. There are several approaches for an analysis of this kind of contact problems [4, 8]. One of the most efficient and useful ways for the numerical modelling of interface damage, especially the crack initiation and crack propagation, is by applying of the cohesive-zone models.
The Cohesive Interface Model (CIM), is introduced and implemented in a Symmetric Galerkin BEM (SGBEM) code. The regularization is proposed so that convex quadratic energy functionals are obtained and quadratic programming algorithms can be efficiently applied. In the following sections the developed model is described, its numerical solution is outlined and an example is solved to assess its applicability in the area of laminate composites.

Excerpt from ISBN 978-3-00-053387-7

2. Cohesive interface models

2.1. Description of the interface model

For the sake of simplicity and notation, only two debonding 2D bodies will be considered in the description of the model. Let us consider a layered structure defined by a planar domain $\Omega^\eta \subset R^2 \ (\eta = A, B)$ with bounded Lipschitz boundaries $\partial\Omega^\eta = \Gamma^\eta$, Fig.1. However, it should be noted that the theory can be generalized to allow also 3D bodies. Let us denote the unit outward normal vector n^η defined at Γ^η and similarly we can denote the unit tangential vector s^η such that it defines anti-clockwise orientation on Γ^η.

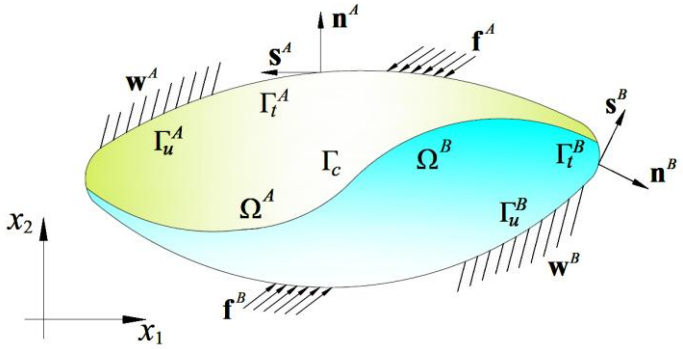

Figure 1. Two debonding domains and the used notation.

The contact zone Γ_C is defined as the common part of Γ^A and Γ^B, i.e. $\Gamma_C = \Gamma^A \bigcap \Gamma^B$. The Dirichlet and Neumann boundary conditions are defined on the outer boundary parts, respectively, prescribing displacements $u^\eta = w^\eta$ on Γ_u^η and tractions as $t^\eta = f^\eta$ on Γ_t^η. It is considered that the subdomains can debond along the interface Γ_C. The interface is considered as an infinitesimally thin layer of an adhesive. Interface damage mechanism is modelled by a scalar damage variable ζ, which has meaning of the level of interface damage. ζ varies at each interface point between one and zero: $\zeta(x) = 1$ corresponds to the undamaged state, $\zeta(x) = 0$ reflects total rupture.

2.2. Bilinear cohesive interface model

In engineering and computational mechanics practise is often preferred bilinear response of the interface model, including the so-called softening period. This kind of model is referred to as bilinear Cohesive Interface Model (CIM) [8]. Bilinear CIM with softening period has been found to approximate accurately Mode I experimental results. Considering, e.g., pure opening Mode I, the failure of a layer point occurs when the driving force G reaches the activation threshold, usually reffered to as fracture energy, G_d, and correspondingly both, the mechanical stress t_n and relative normal displacement $[u]_n$, achieve their critical values, respectively.

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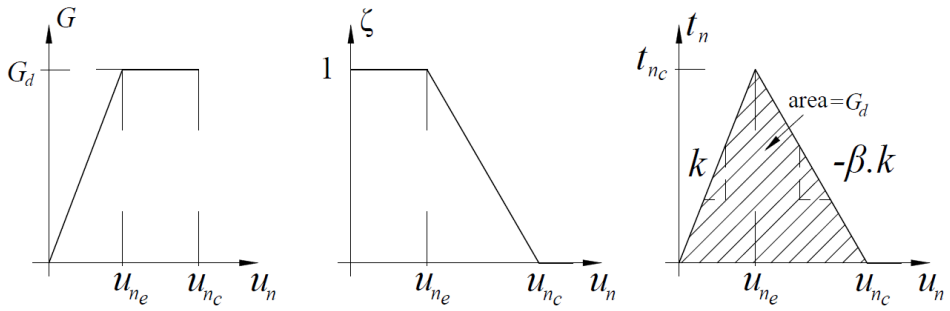

Figure 2. CIM response for the driving force G, the damage parameter ζ and mechanical stress σ.

Achieved softening period depends on a constant β [7], Fig.2 c.), defined as $\beta = \frac{u_e}{u_c - u_e}$. The mechanical stress follows the law $t = k(\zeta)\cdot[u]$, where k is the interface stiffness. For normal sitffness holds $k(\zeta) = k_n\left(\frac{\beta\cdot\zeta}{1+\beta-\zeta}\right)$. In the first, linear-elastic part of the stress-relative displacement diagram, can be observed that the stress is a linear function of $[u]_n$ and holds $t_n = k_n\cdot[u]_n$, up to achieving its critical value $t_{nC}^2 = k_n\ \frac{2G_d\cdot\beta}{(1+\beta)}$, whereas in the second, softening part of diagram, the stress evolves linearly until it vanishes for the critical relative displacement $u_{nC}^2 = \frac{2G_d\cdot(1+\beta)}{k_n\cdot\beta}$.

3. Mathematical concept of the delamination process

This section reviews the mathematical formulation of the energetic approach of interface failure mechanism for CIM. The solution is acquired by variational formulation, which exploits developed numerical treatment of inelastic process.

3.1. Energetic formulation of the interface damage

To define the energetic conception of the interface damage mechanism, let us consider the stored energy functional [4, 8, 9] in the anisotropic structure obeying the aforementioned type of the interface damage in a generalized plain-strain state [6]. The stored energy functional is defined as:

$$E_C(\tau,u,\zeta) = \int_{\Omega^A}\frac{1}{2}\varepsilon^A : C^A : \varepsilon^A\, d\Omega + \int_{\Omega^B}\frac{1}{2}\varepsilon^B : C^B : \varepsilon^B\, d\Omega + \int_{\Gamma_C}\frac{1}{2}\left[k_n(\zeta)\,[u]_n^2 + k_s(\zeta)[u]_s^2 + k_z(\zeta)[u]_z^2 + k_g([u]_n^-)^2\right]d\Gamma, \tag{1}$$

with the admissible displacements $u^\eta = w^\eta(\tau)$ on Γ_u^η and the small strain tensor $\varepsilon^\eta = \varepsilon(u^\eta)$, the potential energy of external forces:

$$F(\tau,u) = -\int_{\Gamma_t^A} f^A.u^A d\Gamma - \int_{\Gamma_t^B} f^B.u^B d\Gamma. \tag{2}$$

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And the dissipation potential:

$$R(\dot{\zeta}) = -\int_{\Gamma_C} G_d \, \dot{\zeta} d\Gamma \quad \text{for} \quad \dot{\zeta} \le 0 , \tag{3}$$

as parameter ζ has only decreasing character during the process of interface damage.

3.2. Numerical solution and computer implementation

The numerical procedures proposed for solving the aforementioned problem contain time and spatial discretizations, as usual. They are considered separately. The time discretization includes a semi-implicit scheme in order to provide a variational structure to the solved problem [4]. The procedures are developed so that the solved problem is formulated in terms of the boundary data only, with the spatial discretization carried out by SGBEM. The expressions for the model in the displacement variables are such that they allow quadratic implementation.

3.2.1. Time discretization

The time-stepping scheme is defined by a fixed time step size τ_0 such that $\tau^k = k\,\tau_0$ for $k = 1,\ldots T/\tau_0$. The rate of damage can be approximated by $\dot{\zeta} \approx (\zeta^k - \zeta^{k-1})/\tau_0$, where ζ^k denotes the solution at the discrete time τ^k. The differentiation with respect to the rate of damage can be replaced by the differentiation with respect to ζ.

$$\partial_{\dot{\zeta}} R(\dot{\zeta}) \approx \tau_0 \, \partial_{\zeta^k} R\left(\frac{\zeta^k - \zeta^{k-1}}{\tau_0} \right). \tag{4}$$

It means that the inclusion is approximated at discrete times τ^k by the first order optimality conditions for the energy functionals H_u^k and H_ζ^k :

$$H_u^k(u) = E(k\tau_0, u, \zeta^{k-1}) + F(k\tau_0, u), \tag{5}$$

$$H_\zeta^k(\zeta) = E(k\tau_0, u^k, \zeta) + \tau_0 R\left(\frac{\zeta - \zeta^{k-1}}{\tau_0} \right), \tag{6}$$

Specifically, assuming ζ^{k-1} and u^{k-1} known from the previous time step, the problem includes two subsequent minimisations:

- First minimize $H_u^k(u)$ with $u^k = \arg\min H_u^k(u)$.
- Second minimize $H_\zeta^k(\zeta)$ with $\zeta^k = \arg\min H_\zeta^k(\zeta)$.

The recursive time-stepping procedure naturally starts from the solution at $k = 1$ calculated by using the initial conditions $u^0 = 0$ and $\zeta^0 = 1$. The optimality solution is denoten by (u^k, ζ^k) [2].

3.2.1. Spatial discretization and SGBEM

The role of the SGBEM in the present computational procedure is to provide a complete boundary-value solution from the given boundary data for each domain in order to calculate the elastic strain energy in these domains. Thus, the SGBEM code calculates unknown tractions along $\Gamma_C \cup \Gamma_u^\eta$ and unknown displacements along Γ_t^η, assuming the displacaments jump at Γ_C to be known from the used

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minimization procedure, in the same way as proposed and tested in [9] for the solution of the presented Boundary Value Problems an SGBEM approach has been applied .

4 . Numerical example

The present work is focused on the interlaminar fracture toughness test of composite materials. For numerical example, Double Cantilver Beam (DCB) is used. The test is used in aeronautical industry to evaluate the interlaminar fracture toughness in composite-composite joints are performed by well known standard procedures [1, 5].
The presented formulation of interface contact problem has been tested numerically by a computer code, which was implemented in MATLAB. The developed numerical algorithm exploits the variationally based Symmetric Galerkin Boundary Element Method to calculate the elastic solution at the interface and in each subdomain. The geometry in the presented example includes two domains. The geometry of model and load configuration is conspicuous from Fig. 3.

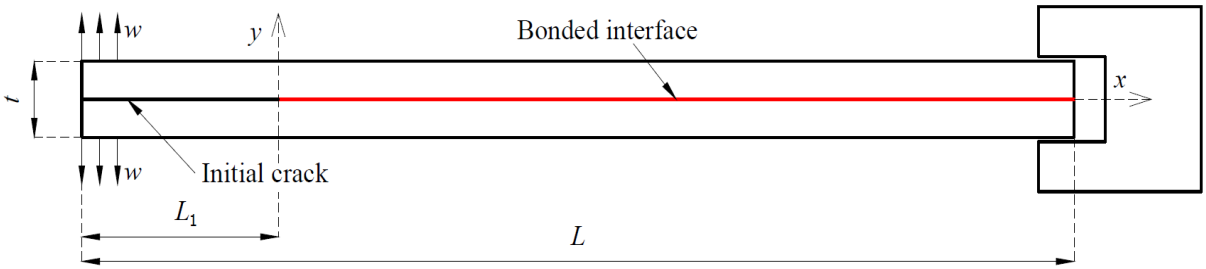

Figure 3. Scheme of DCB specimen numerical model.

4.1. Model properties

In this example, a simulation of the interlaminar fracture toughness test is going to be carried out. The laminate considered is an 8552/AS4 carbon fibre – epoxy composites (having only 0º plies), with the following orthotropic properties: $E_x = 135$ GPa, $E_y = 10$ GPa, $E_z = 10$ GPa, $G_{xy} = 5$ GPa, $G_{xz} = 5$ GPa, $\nu_{xy} = 0.3$, $\nu_{yz} = 0.4$, $\nu_{xz} = 0.3$.
The properties estimated for bilinear CIM are: $k_n = 500$ GPa/m, $\sigma_c = 15$ MPa, constant $\beta = 0.429$ and $u_{nc} = 0.1$ mm corresponding to a value of $G_{Ic} = 750$ Jm^{-2}, see Fig.2 [1, 5].
The applied loading w is assumed on the both laminates in the direction normal to the specimen boundary. In total, there were considered $k = 250$ load steps. The loading process defines the prescribed displacements increasing during the loading process: $w^k = ut^k$ for $k = 1,2,..250$ $u = 0.01$ mm and $t^k = kt_0$, $t_0 = 1$ s. The dimensions of numerical model are : $L = 195$ mm, length of initial crack $L_1 = 55$ mm and $t = 10$ mm. Due to the symmetry of the problem there are not tangential relative displacements.

4.2. Results

The achieved numerical solution of the investigated interface contact model is presented in Fig. 4.

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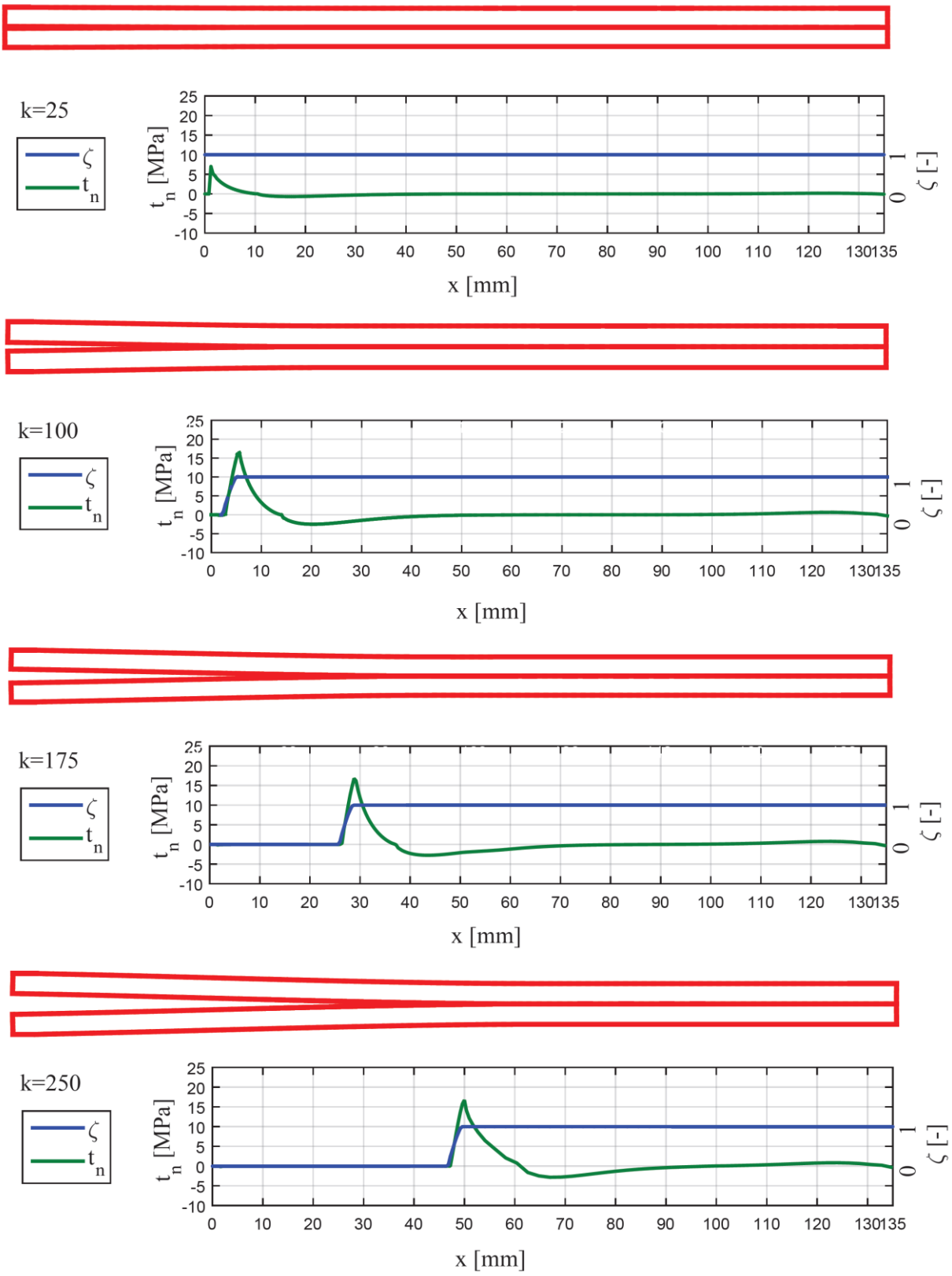

Figure 4. Numerical deformed shapes, response of the normal tractions and of damage parameter at the interface at the pertinent loadstep k.

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The graphs present the evolution of the damage which provokes an interface crack of a layered anisotropic structure. In the time step $k = 25$ can be observed undamaged interface, damage parameter $\zeta(x) = 1$ for $x \in \langle 0;135 \rangle$, because normal tractions t_n have not achieved their critical value $t_{nc} = 15$ MPa. There can also be observed the distribution of the normal tractions in the vicinity of the crack tip. The pertinent deformation is plotted directly above the graphs for each selected time step *k*. The numerical model confirms the expected behaviour in accordance with the applied theory and asseses its applicability for layered anisotropic materials.

6. Conclusions

A numerical study of the stress quantities in a damaged interface of a layered anisotropic structure has been carried out and an energy-based model for solving the contact problem has been discussed. The SGBEM has proved to be a very suitable and accurate tool to deal with the problem under consideration. The proposed cohesive zone approach was obtained by considering a new two-parameter damage dependent interface stiffness function which provides the required bilinear dependence if stress and strain quantities. Observed results shows the potential of numerical model and its applicability for more complicated numerical examples.

Acknowledgments

The authors acknowledge the financial support from the Grant Agency of Slovak Republic VEGA. The project numbers are 1/0078/16 and 1/0477/15.

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Excerpt from ISBN 978-3-00-053387-7