BRIDGING SCALES IN FLUID-SOLID-POROUS MODELLING APPROACHES OF INFUSION-BASED PROCESSES IN INDUSTRIAL CONTEXT

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Abstract
Infusion-based processes are promising routes for manufacturing primary structures in out-of-autoclave processes. However, no model permits yet to finely tune such processes in industrial environments. From a scientific point of view, modelling these processes requires well-founded numerical approaches to represent the infusion of a thermo-reactive fluid into / out of orthotropic non-linear preforms undergoing finite strains, at various scales of observation. In past works, we have extensively developed numerical approaches at the macro scale to couple resin flows in the distribution medium (Stokes) placed on a side of the preform stacking, and through the fibrous preforms seen as orthotropic porous media (Darcy) undergoing finite strains. Since we are deeply convinced that predicting voids formation and the whole infusion process cannot be properly carried out at the macroscale, we have more recently considered the local scale of the fibre network where impregnation may be modelled as a bifluid-solid contact model involving capillary effects. Bridging both macro and micro-scale approaches is the ultimate aim of the work under progress initiated in the Hexcel Chair for "Advanced Numerical Modelling of Infusion-based Processing for New Generation Composite Structures" at Mines Saint-Etienne which sets multi-scale coupled fluid/solid/porous mechanics approaches relying on high-performance computing.

1. Introduction

Liquid Composite Moulding (LCM) processes, such as the Liquid Resin Infusion (LRI) process or Vacuum Assisted RTM (VARTM), are in constant development in high performance applications to propose more efficient ways (cost, cycle times, void content...) to produce large dimension structures. Especially suitable for aeronautics, this type of process consists in a compaction of a dry reinforcement lay-up of preforms placed in a half mould sealed by a vacuum bagging film (Figure 1 left). Under the vacuum depression effect, the stacking is compacted while a liquid resin fills in a stiff distribution medium placed under/over the preform to create a resin-feeding bed. Then, the resin infuses across the thickness of the layer stack which swells simultaneously under the inner fluid resin pressure effect. When the resin has infused the dry reinforcement, the resin feeding inlet is cut-off and the cure stage begins to freeze the final piece dimensions. Despite numerous advantages, the control of these processes is difficult in terms of final dimensions or filling-time predictions.

The simulation of LCM processes may involve multiple physical phenomena: fluid mechanics, solid

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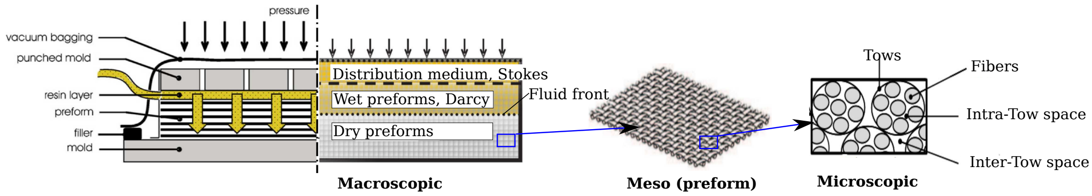

Figure 1. The resin infusion process - schematic and corresponding 3 zones domain decomposition at macro scale, and meso-micro scales.

mechanics, thermal effects and resin cross-linking [1]. Today, the strongest bottle-neck in understanding these processes corresponds to the strong interaction between the reinforcement deformability and the resulting fluid flows. Moreover, even if one considers 'only' the fluid-solid mechanics problem, *i.e.* isothermal non-reactive conditions, these phenomena can be described at various scales (Fig. 1): the microscopic scale, which describes resin flows between the fibers; the mesoscopic scale where resin flow between the tows can be simulated; and the macroscopic scale, which corresponds to the scale of the piece. The first objective of our ongoing work is to propose a robust approach for the simulation of theses processes, at both macro and micro scales.

Historically, as a response to engineers, the first modelling of this process has been settled at the macroscopic scale, *i.e.* where preforms are seen as orthotropic homogeneous equivalent media. At this scale, to be representative of the resin flows through specific pipes / distribution media and through the porous preform, simulations have to consider the coupling between flows in a porous medium (fibrous reinforcement) and non-porous medium (channels and flow media) while taking into account large mechanical deformations of the porous medium. During the past years, we have developed and enriched continuously a finite element framework based on coupling flows in a purely fluid region, ruled by Stokes' equations, and in a porous region governed by Darcy's equations [1, 2]. This model is based on a strongly coupled Stokes-Darcy flow associated with a Lagrangian mechanical formulation considering the large deformation of preforms during the process. This numerical model was then applied in an industrial framework, characterized by reinforcements with anisotropic properties (very low permeability and stiffening mechanical behaviour), to predict filling times, piece dimensions and fibre fractions [3, 4]. This macro scale approach will be presented in the first part of this short paper.

Finely tuning LCM processes is now the next step forward. It is known that critical mechanisms driven by very local effects, such as permeability or micro/macro-void (Figure 1) creation which can significantly affect the quality of composite parts have to be tackled. A strong effort has to be made to properly understand the infiltration of the liquid resin into the intrinsically multi-scale porous fibre network. This infiltration is mainly driven by the intimate contact between the infiltrating fluid and the inhomogeneous micro-structure of the preform. Providing an accurate characterization of phenomena at the microscopic scale within a finite element framework is hence mandatory. It involves modelling the two-phase capillary flows (air/resin) and the wetting effect between the fluids and the fibrous solid, which are determinant for the creation of micro/macro-voids and their subsequent evolution [5]. This recent work will be shortly presented in the second part of this paper.

2. Coupled fluid-porous problem at the macro scale

A vast literature has been dedicated to numerical methods for solving the Stokes-Darcy coupled problem [1], either relying on multi-domain (decoupled) approaches [3, 6] using iterative methods to equilibrate quantities involved through their interface, or based on monolithic approaches consisting in using one

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single mesh to solve both Stokes and Darcy equations [2, 7]. This latter considers naturally a strong coupling at the Stokes-Darcy interface and the robustness of this approach can be ensured by using specific stabilization techniques [8, 9].

As stated earlier, during the process, there is a mutual influence of the resin on the preform behaviour and of the preform deformation on its permeability. The presence of resin in the wet preforms can be classically accounted for through the Terzaghi's law. Conversely, the mechanical deformation, modifying the permeability, is considered naturally through a local modification of the porosity which derives from the transformation gradient solution of the mechanical equilibrium [1]. One of the main features of the present approach is to couple a unified fluid formulation with a non-linear solid mechanics problem./ This leads to a robust formulation for the simulation of manufacturing processes, even in industrial conditions with real and severe physical properties such as low and anisotropic permeabilities, or thin and complex shapes.

2.1. Stokes-Darcy mathematical model

Let us define the computational domain $\Omega \subset \mathbb{R}^m$ $(m = 2, 3)$, as a bounded domain made up of two non-overlapping sub-domains Ω_s and Ω_d separated by an interface $\Gamma = \partial\Omega_s \cap \partial\Omega_d$.

The fluid flow in Ω_s is governed by the Stokes' equations which classically writes :

$$\begin{array}{rcl} -\boldsymbol{\nabla} \cdot (2\eta\dot{\boldsymbol{\varepsilon}}(\mathbf{v}_s)) + \boldsymbol{\nabla} p_s & = & \mathbf{f}_s \\ \boldsymbol{\nabla} \cdot \mathbf{v}_s & = & 0 \end{array} \tag{1}$$

where $\mathbf{v}_s$ and p_s are the fluid velocity and pressure in the Stokes domain whereas $\boldsymbol{\nabla}$ and $\boldsymbol{\nabla}\cdot$ represent the spatial gradient and divergence operators. $\dot{\boldsymbol{\varepsilon}}$ is the strain rate tensor defined by $\dot{\boldsymbol{\varepsilon}}(\mathbf{v}_s) = \frac{1}{2}(\boldsymbol{\nabla}\mathbf{v}_s + \boldsymbol{\nabla}^\top \mathbf{v}_s)$, and $\mathbf{f}_s$ are the volumetric forces. This set of equations is completed with the proper boundary Neumann and Dirichlet conditions.

The flow of an incompressible fluid through a porous medium Ω_d can be described by the Darcy's equation which writes - $\mathbf{v}_d$ is the Darcy's velocity (macroscopic mean velocity), p_d the fluid pressure:

$$\begin{array}{rcl} \boldsymbol{K}^{-1}\eta\mathbf{v}_d + \boldsymbol{\nabla} p_d & = & \mathbf{f}_d \\ \boldsymbol{\nabla} \cdot \mathbf{v}_d & = & 0 \end{array} \tag{2}$$

where $\boldsymbol{K}$ is the permeability tensor which can be reduced to a scalar in an isotropic case, $\mathbf{f}_d$ are the volumetric forces. Proper Neuman and Dirichlet conditions close the problem.

Between the Stokes and Darcy domains, some specific conditions have to be considered on the interface Γ: 1/ the mass conservation expressed by the continuity of the normal velocity field $[\mathbf{v}_s - \mathbf{v}_d] \cdot \mathbf{n}_s = 0$; 2/ the continuity of the normal stress $\mathbf{n} \cdot [\boldsymbol{\sigma}_s - \boldsymbol{\sigma}_d] \cdot \mathbf{n} = 0$; 3/ the continuity of the tangential velocity considered through a Beaver Joseph Saffman condition [10] which allow to control the tangential velocity on the interface $2\mathbf{n} \cdot \dot{\boldsymbol{\varepsilon}}(\mathbf{v_s}) \cdot \boldsymbol{\tau_i} = -\frac{\alpha}{\sqrt{K_{ii}}}(\mathbf{v_s} \cdot \boldsymbol{\tau_i})$ where α is a dimensionless parameter, called slip coefficient, $\boldsymbol{\tau_i}$ $(i = 1, ..., m-1)$ are the unit tangential vectors on the interface and K_{ii} the permeabilities related to the tangential directions.

From this mathematical problem, a weak form can be deduced and finite elements formulated along. It is extensively presented in [11] and extended to orthotropic porous media in [4]. In few words, both velocity and pressure fields are approximated by continuous piecewise functions. Stability of the discrete Stokes-Darcy coupled problem is subsequently ensured by using a Variation Multi-Scale (VMS) stabilization method [8, 9]. In the monolithic approach considered here, both the interface Γ separating the Stokes

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and Darcy domain, and Γ_f defining the moving flow front are not described exactly by a set of elements. Rather, these interfaces are defined thanks to two level-set functions. In few words, *Level-set* techniques consist in defining a signed function which may represent the distance to the interface (the interface is then represented by the iso-value zero). In order to improve the stability and the computation of quantities on interfaces, special integration rules are used in the elements cut by the interfaces [4, 12, 13]. The main idea is to subdivide, with cheap numerical operation, the elements intercepted by the interfaces into sub-elements but only for integration enrichment purpose. This so-called Surface Local Reconstruction (SLR) methodis purely geometric, and does not change the number of degree of freedom of the problem.

2.2. Solid mechanics for the preform response

During the process, wet preforms undergo finite strains due to both the exxternal mechanical compaction exerted by atmosphere, and swelling caused by to the resin pressure effect. In order to model some complex shape parts processing simulations, a holistic solid mechanics frame was coupled to the Stokes-Darcy problem presented above. One of the key feature of the approach developed is that no hypothesis has to be made regarding the porosity change during compaction and infusion, since the Jacobian of the transformation given by the solid mechanics problem solution yields the volume change [1]. Then, permeability, which strongly depends on the fibre fraction evolution, can be updated accordingly at any material point of the domain [14] based for known porosity-permeability relationships. Conversely, the fluid-structure problem is weakly coupled using a sequential algorithm in which both fluid and solid problems are solved considering the last converged results of the other problem. Then, in the solid mechanics problem, the effect of the resin on the non-linear preform behaviour is accounted for through a Therzaghi's law where the Cauchy stress tensor in the preform $\boldsymbol{\sigma}$ is the sum of the effective stress corresponding to the mechanical response of the fibres $\boldsymbol{\sigma}_{eff}$ plus the hydrostatic fluid pressure possibly weighted by saturation $sp\mathbf{I}$: $\boldsymbol{\sigma}(\boldsymbol{\epsilon}) = \boldsymbol{\sigma}_{eff}(\boldsymbol{\epsilon}) - sp\mathbf{I}$.

Using a Lagrangian based formulation, well adapted to represent the non-linear behaviour of the porous medium in a finite strain framework, the problem can be classically written. The material considered in this study presents a non-linear behaviour characterized experimentally by a compression test in dry conditions. Considering the mechanical deformations of preforms during the process, the in-plane mechanical properties can be considered, as a first approximation, as linear and defined with classical elastic properties. On the contrary, since the non-linear transverse behaviour of a fibrous preform is essentially due to local fibre re-arrangements, the transverse Poisson ratios are closed to zero and can be approximated by a null value. It allows to represent rather directly the transverse non-linear behaviour given by some experimental measurements [1].

2.3. Simulation of a "T" shape stringer infusion

As an illustration, here is considered the infusion of a stiffened panel usually found in aeronautics, and characterized by strong curvatures and thickness variations (Figure 2(a)) [4]. The simulation has been performed by using the finite element software Z-set [2, 15]. The computational domain is divided into one purely fluid-domain (flow medium) and three otrhotropic porous domains - local coordinates system $\mathcal{R}(O, X_I, X_{II})$ - representing respectively the stringer, the skin and the filler (Figure 2(a)). Dimensions are given in Figure 2(a) and symmetry conditions are used to reduce the size of the numerical model. On this figure, the red dashed line represent the Stokes-Darcy interface. Physical properties used are representative of industrial conditions (Figure 2(a)). The transverse permeability is initially closed to $K_{II} = 1.10^{-15}\ m^2$ but changes with the fibre fraction. Both permeability and the mechanical transversally isotropic response are defined in the structural frame $\mathcal{R}$. The flow is classically taken as a stiff isotropic elastic medium. A fluid pressure differential of $1.10^5\ Pa$ is prescribed between bottom and top faces of the domain (corresponding respectively to the resin inlet and outlet) whereas the remaining

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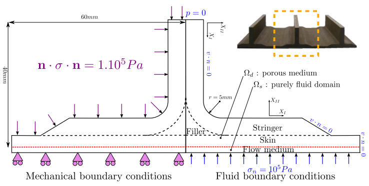
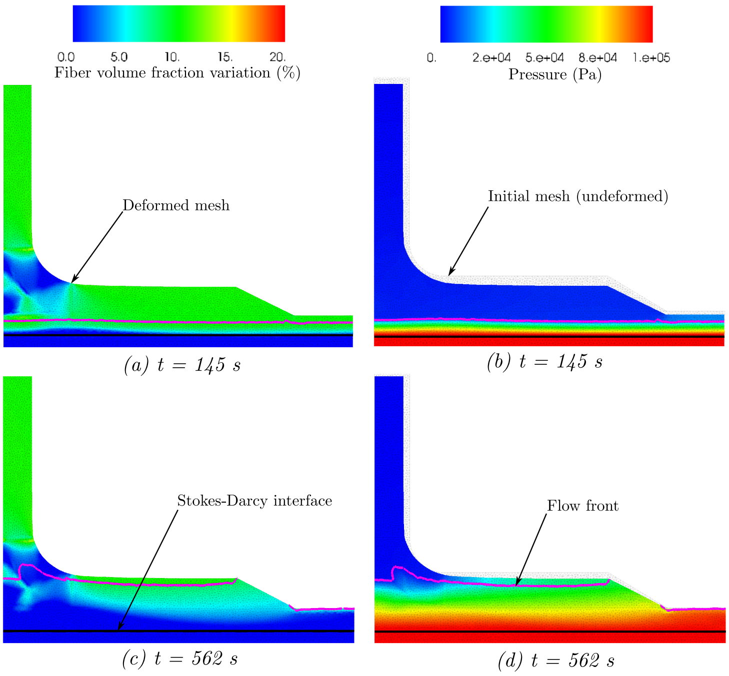
boundaries are considered as impervious walls (fluid boundary conditions in blue Figure 2(a)). Displacements are null on the bottom face and a following normal stress of 1.10^5 Pa is prescribed on the remaining boundaries to represent the atmospheric pressure effect.



(a) Boundary conditions; air viscosity $\eta_a = 1.10^{-5}$ $Pa.s$, resin viscosity $\eta_r = 3.10^{-2}$ $Pa.s$, in-plane permeability $K_I = 1.10^{-13}$ m^2.



(b) Fiber fraction and pressure fields at 145s (a-b) and 562s (c-d).

Figure 2. Infusion of a "T" shape stiffened panel.

Figures 2(b) present some simulation results at 145 s when only one half of the skin thickness is filled (Figures 2(b)(a-b)), and at 562 s when the horizontal part of the stringer is almost filled (Figures 2(b)(c-d)). Figures 2(b) (b) and (d) show that low permeabilities of the preform, induced by the initial compaction stage, introduce a pressure gradient only in the wet area of the porous domain. The evolution of the resin pressure leads to significant modifications in fibre volume fraction according to Figures 2(b) (a) and (c) due to swelling. Globally the resin pressure introduces an important and non-homogeneous volume fraction changes in the piece leading to non homogeneous permeability in the preform. Considering the permeability evolution of the material, a 10% of fibre fraction variation leads to a permeability twice higher or lower modifying significantly the fluid velocity and filling times. Globally, after filling, fibre fractions are not acceptable for aeronautic parts, and a refine strategy of infusion has to be considered by process engineers.

3. Local scale flows in fibrous network

Modelling infusion processes at the macro scale is inherently limited to phenomena which can be, in some manners, included in parameters such as permeabilty. Indeed, it is well known that saturated and transient permeability differ, in plane and out of plane [14]. This is intrisically due to the local wetting effects which occur at the fiber-fluid level (Figure 3-a) and must, hence, be studied. In the past decades, various CFD techniques have been developed for modelling two-phase flows with moving interfaces. Every approach has its advantages and drawbacks, but face anyway the ill-posed nature of the problem to be solved : determine the simultaneous dynamic contact angle / slip line position change. More descriptions of the existing various methods can be found in literature [16]. As for the LCM applications,

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we choose the Eulerian description for the fluid domain and the *Level-set* method for the interface description allowing accurate description of curved interfaces with strong topological evolutions, relying on the previously valdiated developments for the stabilized Stokes flows simulations. However, the accurate simulation of capillary driven flows remains a challenge, mainly due to the singular surface tension force acting on the interfaces, and the associated discontinuities of pressure and viscosity. Moreover, spurious velocities can be observed in the vicinity of the interface when considering a standard finite element approximation in Eulerian approach, which may significantly affect precision and robustness of numerical simulations. Most recent researches focus on the improvement of existing methods.

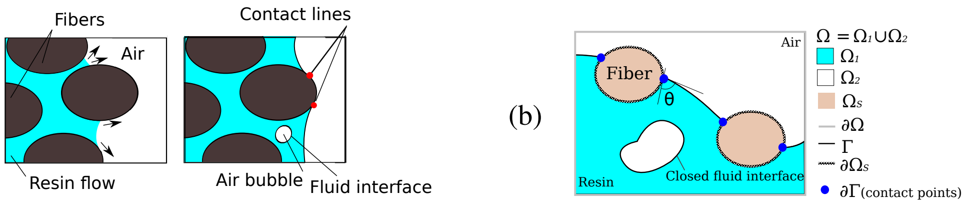

Figure 3. Moving interfaces at the microscopic scale (a) and corresponding model description (b).

3.1. Tensorial formulation of the surface tension contribution

For the simulation of both capillary driven flow - where the tension surface effects predominate-, and wetting phenomena - where surface tension and viscosity effects co-exist-, observed in the LCM processes simulation, we have proposed [5] a robust Eulerian finite element method based on a careful computation of the driving force term acting on the fluid-fluid interface $\int_\Gamma \gamma\, \kappa\, \mathbf{n}\, dS$ - where γ is the surface energy and κ is the local curvature. An alternative tensorial method [13, 17] was also implemented to avoid the explicit computation of the curvature (second derivative of the *Level-set* function), which is tricky when the *Level-set* function is approximated with piecewise linear functions. Therefore, the surface tension term contribution can be computed in the weak form of the problem:

$$\int_\Gamma \gamma\kappa\mathbf{n}\cdot\mathbf{w}dS = \int_\Gamma \gamma[\mathbb{I} - \mathbf{n}\otimes\mathbf{n}] : \nabla\mathbf{w}dS \qquad (3)$$

which holds only when the surface Γ is closed [13] - $\mathbf{w}$ is the virtual velocity field. The SLR method presented above [13] was considered for computing this term, and proved to be efficient.

3.2. Boundary conditions for wetting phenomena

The contact line is the line of intersection between two fluids and a fiber surface $\partial\Omega_S$, it is represented by the two blue dots in the 2D situation of Figure 3-b. The boundary conditions for the wetting phenomena allow to close the line dynamic problem which consists in the simultaneous description of the contact angle and the slip of the contact line on the fiber surface. A major challenge in the simulation of the contact line dynamic is that the classical hydrodynamic equations, such as the Stokes' equations, coupled with the conventional no-slip boundary condition leads to a stress singularity at the contact line [18]. Various methods have been proposed recently to remove the stress singularity, such as the diffuse interface method or methods replacing the no-slip boundary condition by some slip model in the vicinity of the contact line, such as early suggested by Navier. As for the contact line dynamic, one can use an iterative method to take into account the contact angle at the fluid/fiber interfaces $\partial\Omega_S$.

This method was successflully applied to represent various dynamic cases which can be found in [5]. Here is presented, in Figure 4, the case of a drop which is formed in contact with a substrate. The contact angle is the target of the simulation in order to represent properly wetting and non-wetting characters.

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One can observe the pressure field discontinuity properly computed in both fluids around the drop surface thanks to specific methods [13]. Work is under progress to strengthen this approach in fiber networks.

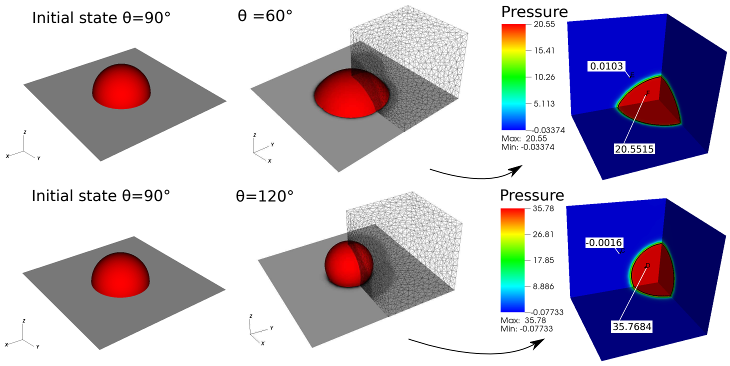

Figure 4. Drop evolution with a contact angle of 60 ° (top) and 120° (bottom) over a plane substrate, γ = 0.03 $N.m^{-1}$, viscosity of fluids: 30 $mPa.s$ (drop) and 0.3 $mPa.s$ (surrounding fluid).

4. Conclusion

A numerical strategy has been proposed for the simulation of resin infusion based processes. In these processes, it exists a strong interaction between the fluid flow and the porous domain deformation. The relevance of a fluid monolithic approach for the coupling to a solid mechanics problem has been demonstrated on the simulation of a ”T” shape stiffened panel. Simulation results have shown the robustness of the tools which appeared very stable even on complex simulation case presenting some characteristic difficulties such as strong curvatures, thickness variations, but also sharp variations of the mechanical and morphological properties. The second part of the approach is dedicated to surface tension driven problems which locally control the resin infiltration, and therefore the preforms permeability as well as the micro-macro voids formation.

Current work is under progress on both scales, and bridging both macro and micro-scale approaches is the aim of the Hexcel Chair for ”Advanced Numerical Modelling of Infusion-based Processing for New Generation Composite Structures” at Mines Saint-Etienne which sets multi-scale coupled fluid/solid/porous mechanics approaches relying on high-performance computing.

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